V. Considerations on Membership Functions

V-1. Remaining Issues

There is no such thing as a membership function. There is only a function in the software that determines membership score. In mathematics, a function refers to a mechanism that represents the relationship between two things, often expressed in formulas. It's like a vending machine where you input some value (money), specify the calculation conditions (press the button for the desired item), and get some value (product) in return. This mechanism is called a function.

The determination of membership scores does not follow any established mechanism; it can be decided based on the analyst's purpose, existing information, and experience. Such a thing is not called a function because there is no mechanism. What exists is just the determined membership score. The process of determining this membership score is called calibration. They say that natural scientists frequently perform calibration. Indeed, serious natural scientists frequently perform calibration. Nowadays, the accuracy of analytical machines has improved, and many analytical instruments have built-in calibration functions, so frequent calibration may not be necessary. In the past, when analytical instruments did not have such convenient functions, calibration was performed by analyzing standard samples. When the commentator was younger and engaged in experimental science, he often adjusted analytical instruments. Scientific balances were not so stable in sensitivity, so the sensitivity of the balance was adjusted monthly using standard weights. This is calibration. Calibration is the process of adjusting to ensure objectively correct measurements using standard samples with known concentrations or weights. What they are doing is different from this. They are transforming data to suit their analysis, which is tuning, not calibration. Changing the format of data or transforming data for the desired analysis is called tuning. Ethically, tuning is a rather risky task for researchers. If the analyst has a predetermined conclusion they want to reach and tunes the data to fit that conclusion, it is clearly data falsification and violates research ethics. Removing outliers is also a form of tuning, but whether to do it or not always troubles the analyst. The tuning process must be explained and recorded in a way that others can understand. Probably, because the term "data tuning" has a negative image, they used the term "calibration," but this is a complete misunderstanding of the term. What they are doing is tuning. If it is tuning, they should show the necessity, the transformation method, the original data, and the results. Since they do not show this, their tuning is bad tuning.

That aside, various methods for tuning membership score can be considered. They seem to use cumulative probability distributions used in numerical analysis when determining membership scores. While they say it is not probability, using existing probability distributions does not feel logically inconsistent, but since no other suitable method comes to mind, this must be accepted. The arbitrary determination of the median makes one want to question its basis. Probably, without doing so, the analysis would not go well, so an analysis using cumulative probabilities of a normal distribution centered on the mean is attempted.

2-1. Membership Score Using Symmetric Probability Distribution (Normal Distribution)

Table 29 shows a comparison of membership values when the analyst arbitrarily assigns the origin at a cumulative probability of 0.50 and when assuming a normal distribution and using the cumulative probability distribution values as they are. As the origin shifts, the horizontal spread of the distribution also varies, making it difficult to capture the overall differences. However, in A, the membership scores for CZE and FIN, in C for POL, and in E for FIN and ITA, have increased from below 0.50 to above 0.50 due to "calibration." Conversely, in B and D, the membership values for ESP and ITA have decreased from above 0.50 to below 0.50. Overall, the differences were minimal for PRT, UK, NLD, and ROU, while they were significant for HUN, IRL, and EST.

Table 29. Comparison of membership scores after (calb) and before (norm) moving 0.50 point

	A		B		C		D		E	
Case ID	calb	norm	calb	norm	calb	norm	calb	norm	calb	norm
AUT	0.81	0.62	0.12	0.36	0.99	0.77	0.73	0.66	0.43	0.41
BEL	0.99	0.96	0.89	0.86	0.98	0.70	1.00	0.96	0.97	0.84
CZE	0.58	0.42	0.98	0.94	0.98	0.73	0.90	0.77	0.91	0.72
EST	0.17	0.25	0.07	0.27	0.98	0.71	0.01	0.10	0.91	0.72
FIN	0.58	0.42	0.04	0.17	0.99	0.78	0.09	0.28	0.58	0.49
FRA	0.97	0.90	0.03	0.16	0.98	0.74	0.80	0.70	0.95	0.78
GER	0.89	0.72	0.78	0.81	0.99	0.77	0.96	0.84	0.31	0.33
GRC	0.04	0.17	0.10	0.32	0.13	0.09	0.36	0.48	0.43	0.41
HUN	0.08	0.20	0.17	0.42	0.88	0.51	0.08	0.27	0.13	0.20
IRL	0.72	0.53	0.05	0.21	0.98	0.71	0.01	0.11	0.95	0.78
ITA	0.34	0.32	0.10	0.33	0.42	0.25	0.47	0.53	0.58	0.49
NLD	0.98	0.92	1.00	0.98	0.99	0.80	0.94	0.82	0.99	0.92
POL	0.02	0.13	0.18	0.44	0.59	0.34	0.00	0.06	0.00	0.01
PRT	0.01	0.11	0.02	0.10	0.01	0.01	0.12	0.31	0.01	0.02
ROU	0.01	0.12	0.04	0.17	0.17	0.11	0.01	0.07	0.84	0.65
ESP	0.03	0.15	0.30	0.56	0.09	0.06	0.21	0.39	0.21	0.26
SWE	0.95	0.84	0.13	0.38	0.99	0.80	0.66	0.62	0.91	0.72
UK	0.98	0.93	0.99	0.96	0.99	0.80	1.00	0.97	0.97	0.84

Table 30. Comparison of sorting of countries by fsQCA

	calibration with setting 0.50 point		normal distribution	
set	consist	country	consist	country
A*B*C*D*E	0.904997	UK(0.97),NLD(0.94),BEL(0.89),CZE(0.58)	0.890438	UK(0.80),NLD(0.80,BEL(0.70)
A*b*C*d*E	0.805562	IRL(0.72),FIN(0.58)	0.883436	IRL(0.53)
A*b*C*D*E	0.706196	FRA(0.80),SWE(0.66)	0.863095	FRA(0.70),SWE(0.62)
a*B*C*D*E			0.817518	CZE(0.58)
a*B*c*d*e			0.716418	ESP(0.56)
a*b*c*D*e			0.622024	ITA(0.51)
a*b*C*d*E	0.538335	EST(0.83)	0.746224	EST(0.71)
a*b*C*d*e	0.529202	HUN(0.83),POL(0.59)	0.823708	FIN(0.51)HUN(0.51)
A*B*C*D*e	0.458506	GER(0.69)	0.727829	GER(0.67)
A*b*C*D*e	0.390327	AUT(0.57)	0.768953	AUT(0.59)
a*b*c*d*E	0.288745	ROU(0.53),ITA(0.53)	0.662983	ROU(0.65)
a*b*c*d*e	0.226152	PRT(0.88),ESP(0.70),GRC(0.57)	0.577381	PRT(0.69),POL(0.57)GRC(0.52)

Table 30 compares the differences in country classification resulting from two different "calibrations" (calculation process: Excel sort norm). This table shows the effects of moving the cumulative 0.50 point. The countries that have significantly changed positions are CZE, ESP, ITA, FIN, and POL. These countries originally had relatively low membership scores (just above 0.50) for the given conditions, and their classification easily changes with a shift in the threshold. CZE (Czechoslovakia) moved to the group formed by the UK (United Kingdom), NLD (Netherlands), and BEL (Belgium). ESP (Spain) moved to the group formed by PRT (Portugal) and GRC (Greece), while ITA (Italy) moved to the group of ROU (Romania). FIN (Finland) moved from the group it formed with HUN (Hungary) to the group of IRL (Ireland), and POL (Poland) moved from the group it formed with PRT (Portugal) and GRC (Greece) to the group of HUN (Hungary). In this case, the calibration successfully separated the democratic Finland from the non-democratic Hungary. Overall, the movement of the point has the effect of simplifying the structure by moving ambiguously classified countries into larger groups. Generally, analysis aims to simplify and clarify structures, so if the calibration is done with a valid basis, it is an acceptable tuning. However, whether it is better to simplify or to analyze with a complex structure depends on the case. Further analysis will be conducted using the classification based on the cumulative probability of the normal distribution. (Excel sort norm)

Table 31 shows the countries belonging to the combination of three conditions, with the cumulative probability of the normal distribution as the membership score, and their consistency values.

In the conditions included in the result R (maintenance of democracy), only $A \wedge C \wedge E$ has a consistency of 0.90 or higher. That is

$$A \wedge C \wedge E \longrightarrow R$$

It can be interpreted that if a country is wealthy, has a high level of education, and political stability, democracy is maintained. The countries belonging to this category are Belgium, France, Ireland, the Netherlands, Sweden and the United Kingdom, which are six of the eight countries that maintain democracy, with a coverage of 0.75. Next, $(A \wedge c \wedge E) \vee (A \wedge c \wedge e) = A \wedge$c has a high consistency value, but this is an empty set. In reality, there are no wealthy countries with a low level of education. Following this, the countries $(A \wedge c \wedge E) \vee (A \wedge c \wedge e) = A \wedge$a have relatively high consistency values, but this set includes both democratic countries, Finland and Czechoslovakia, and countries where democracy has collapsed, such as Hungary and Estonia.

Table 31. Consistency and member countries belonging in 3 conditions logical conjunction

⊆R		A	C	E	consis	country
	A*C*E	1	1	1	0.909	BEL(0.70)FRA(0.74)IRL(0.53)NLD(0.80)SWE(0.72)UK(0.80)
	A*c*E	1	0	1	0.885	Ø
	A*c*e	1	0	0	0.878	Ø
	a*C*e	0	1	0	0.804	FIN(0.51)HUN(0.51)
	a*C*E	0	1	1	0.756	CZE(0.58)EST(0.71)
	A*C*e	1	1	0	0.727	AUT(0.59)GER(0.67)
	a*c*E	0	0	1	0.674	ROU(0.65)
	a*c*e	0	0	0	0.525	GRC(0.51)ITA(0.51)POL(0.66)PRT(0.89)ESP(0.74)
⊆r						
	a*c*e	0	0	0	0.966	GRC(0.51)ITA(0.51)POL(0.66)PRT(0.89)ESP(0.74)
	a*c*E	0	0	1	0.935	ROU(0.65)
	a*C*e	0	1	0	0.894	FIN(0.51)HUN(0.51)
	A*c*e	1	0	0	0.872	Ø
	A*C*e	1	1	0	0.847	AUT(0.59)GER(0.67)
	A*c*E	1	0	1	0.775	Ø
	a*C*E	0	1	1	0.773	CZE(0.58)EST(0.71)
	A*C*E	1	1	1	0.428	BEL(0.70)FRA(0.74)IRL(0.53)NLD(0.80)SWE(0.72)UK(0.80)

In the inclusion relationship with r (collapse of democracy), $(a \wedge c \wedge e) \vee (a \wedge c \wedge E) = a \wedge c$ has high consistency, and the consistency of the 2 conditions logical conjunction $a \wedge c$ included in the result r (collapse of democracy) is 0.934, including only the countries where democracy has collapsed: Greece, Italy, Poland, Portugal, Spain, and Romania. Following this, the consistency value of the conjunction $a \wedge C \wedge e$ is high, including Finland and Hungary. The consistency value of this set being included in the collapse of democracy is higher than the consistency value of $A \wedge C \wedge e$, which includes the democratic collapse countries Austria and Germany. From this, it can be considered that Finland maintained democracy despite being likely to become a democratic collapse country under these conditions. The factors behind this should be a subject of further research.

Table 32 shows the analysis results of the inclusion relationship between the logical conjunction of two conditions out of A, C, and E, and the result R (maintenance of democracy). The condition with a high consistency value, which only includes countries that maintained democracy, was $A \wedge E$. The countries that fell under this condition were Belgium, France, Ireland, the Netherlands, Sweden, and the United Kingdom. This is the same as the countries included in the three conditions $A \wedge C \wedge E$, so concluding $A \wedge E \longrightarrow R$ is a more parsimonious expression. Table 33 shows the analysis results of the inclusion relationship with the result r (collapse of democracy). The combinations with high consistency values, which only include countries where

democracy collapsed, were $a \wedge c$ and $c \wedge e$. The countries that fell under these combinations were Greece, Italy, Poland, Portugal, and Spain, with Romania also included in $a \wedge c$. Although $a \wedge e$ has a higher consistency than $c \wedge e$, it cannot be concluded as $a \wedge e \longrightarrow r$ because Finland, a country that maintained democracy, is included. However, this is extremely significant information. Despite the high likelihood that democracy could not be maintained due to economic and political conditions, Finland maintained democracy.

Table 32 Consistency of inclusion of 2 condition logical conjunction in result R and member countries

⊆U	A	C	consist	country
A*c	1	0	0.8909	Ø
A*C	1	1	0.8405	AUT(0.62) BEL(0.70) FRA(0.74) GER(0.72) IRL(0.53) NLD(0.80) SWE(0.72) UK(0.80)
a*C	0	1	0.7573	CZE(0.58) EST(0.71) FIN(0.58) HUN(0.51)
a*c	0	0	0.6741	GRC(0.83) ITA(0.68) POL(0.66) PRT(0.89) ROU(0.88) ESP(0.85)
	A	E	consiste	country
A*E	1	1	0.902	BEL(0.84) FRA(0.748) IRL(0.53) NLD(0.92) SWE(0.72) UK(0.84)
A*e	1	0	0.731	AUT(0.59) GER(0.67)
a*E	0	1	0.641	CZE(0.58) EST(0.71) ROU(0.65)
a*e	0	0	0.492	FIN(0.51) GRC(0.59) HUN(0.80) ITA(0.51) POL(0.87) PRT(0.89) ESP(0.85)
	C	E	consiste	country
A*E	1	1	0.863	BEL(0.70) CZE(0.72) EST(0.71) FRA(0.74) IRL(0.71) NLD(0.80) SWE(0.72) UK(0.80)
a*E	0	1	0.723	ROU(0.65)
A*e	1	0	0.722	AUT(0.59FIN(0.51)) GER(0.67) HUN(0.51)
a*e	0	0	0.545	GRC(0.59) ITA(0.51) POL(0.66) PRT(0.98) ESP(0.74)

Table 33 Consistency of inclusion of 2 condition logical conjunction in result r and member countries

⊆r	A	C	consiste	country
a*c*	0	0	0.934	GRC(0.83) ITA(0.68) POL(0.66) PRT(0.89) ROU(0.88) ESP(0.85)
a*C	0	1	0.788	CZE(0.58) EST(0.71) FIN(0.58) HUN(0.51)
A*c	1	0	0.787	Ø
A*C	1	1	0.477	AUT(0.62) BEL(0.70) FRA(0.74) GER(0.72) IRL(0.53) NLD(0.80) SWE(0.72) UK(0.80)
	A	E	cosister	country
a*e	0	0	0.934	FIN(0.51) GRC(0.59) HUN(0.80) ITA(0.51) POL(0.87) PRT(0.89) ESP(0.85)
A*e	1	0	0.859	AUT(0.59) GER(0.67)
a*E	0	1	0.828	CZE(0.58) EST(0.71) ROU(0.65)
A*E	1	1	0.428	BEL(0.84) FRA(0.748) IRL(0.53) NLD(0.92) SWE(0.72) UK(0.84)
	C	E	consiste	country
c*e	0	0	0.917	GRC(0.59) ITA(0.51) POL(0.66) PRT(0.98) ESP(0.74)
C*e	1	0	0.849	AUT(0.59FIN(0.51)) GER(0.67) HUN(0.51)
c*E	0	1	0.827	ROU(0.65)
C*E	1	1	0.431	BEL(0.70) CZE(0.72) EST(0.71) FRA(0.74) IRL(0.71) NLD(0.80) SWE(0.72) UK(0.80)

Table 34. Consistency of inclusion of single conditions in the results member countries

Inclusion	Consist.	Countries	Inclusion	Consist.	Countries
$A \subseteq R$	0.834	AUTI,BEL,FRA,GER, IRL,NLD,SWE.UK	$a \subseteq r$	0.829	CZE,EST,FIN,GRC, HUN,ITA.POL,PRT, ROU,ESP
$C \subseteq R$	0.779	AUT,BEL,CZE,EST FIN,FRA,GER,HUN IRL,NLD,SWE,UK	$c \subseteq r$	0.843	GRC,ITA,POL,PRT ROU,ESP
$E \subseteq R$	0.786	BEL,CZE,EST,FRA IRL,NLD,ROU,SWE UK	$e \subseteq r$	0.844	AUT,FIN,GER,GRC HUN,ITA,POL,PRT ESP

Looking at the inclusion relationship between a single condition and the result (Table 34), the condition with high consistency, which only includes countries where democracy collapsed, is the low level of education c. The countries that fall under this condition are Greece, Italy, Poland, Portugal, Romania, and Spain, which completely match the combination of two conditions $a \wedge c$ and match five countries with $c \wedge e$. In the combination of two conditions, the conclusion was$A \wedge E \longrightarrow R$. Combining the results of the two conditions, the conclusions are:

$$A \wedge E \longrightarrow R$$

$$c \longrightarrow r$$

These are the conclusions of the analysis using the probability distribution of the normal distribution as the membership value. The number of countries that fall under these conclusions is six countries for $A \wedge E$ (Belgium, France, Ireland, the Netherlands, the United Kingdom) and six countries for c (Greece, Italy, Poland, Portugal, Romania, Spain), totaling 12 countries, with a coverage of 12/18=0.667. Considering that the coverage of the fsQCA conclusion, which was tuned (not calibrated) by selecting the cumulative probability point of 0.50, was 1.00, the range of explanation has significantly decreased. Moreover, the conclusions $A \wedge E \longrightarrow R$ and c⟶r are not complementary. In other words, the tuning resulted in complementary maintenance and collapse conditions, achieving a perfect conclusion with a coverage of 1.00.

The conclusions when using the cumulative probability of the normal distribution as the membership value are:

$$A \wedge E \longrightarrow R$$

$$c \longrightarrow r$$

In everyday language, this translates to: “If a country is wealthy and politically stable, it can maintain democracy. Otherwise, it must strive for better education, or democracy will collapse.” The commentator believes that this conclusion is more meaningful and suggestive than the conclusion obtained from tuning:

$$A \wedge C \wedge E \longrightarrow R$$

$$\widetilde{A \wedge E} \rightarrow r$$

In everyday language, this translates to: “If a country is wealthy, has a high level of education, and is politically stable, it can maintain democracy. Otherwise, democracy will collapse.” Furthermore, highlighting the uniqueness of Finland is a significant achievement. Is increasing coverage so important? Is a perfect complementary relationship necessary? More importantly, it is crucial to discover something through analysis.

V-2-2. Summary of fsQCA

1. In this analysis, we conducted fsQCA by adjusting the median of membership values and fsQCA by using the cumulative probability values of the normal distribution as membership values. We then compared the results.
2. It was found that adjusting the membership values increased the coverage of the conclusions (the proportion of countries that can be explained by the conclusions). In this example, without adjusting the membership values, 12 out of 18 countries could be explained by the conclusions (coverage 0.667), but by adjusting the values, all countries could be explained (coverage 1.000).
3. Without adjusting the membership values, we obtained an asymmetric and non-complementary conclusion: “Rich and politically stable countries can maintain democracy, while countries with low education levels see democracy collapse.” However, the adjusted analysis yielded a complementary and symmetric conclusion: “Rich and politically stable countries can maintain democracy, but countries that are not cannot maintain democracy.”
4. Adjusting the membership values enhances the generality of the conclusions and leads to symmetric conclusions, while the analysis without adjusting the membership values highlights specific cases, albeit with lower generality.
5. When sorting countries into conditions using fsQCA, using consistency as a numerical indicator was somewhat effective, but csQCA was also used to judge the effectiveness of the sorting. In this sense, the effect of quantifying consistency using membership values is low. However, focusing on countries with high membership values that do not match the predicted conclusions can provide new perspectives, which is something that numerical analyses like factor analysis and regression analysis cannot achieve.
6. Comparing consistency values through fsQCA generates various discussions. Considering such possibilities, tuning membership values to increase coverage, simplify conclusions, or enhance complementarity is not very meaningful.
7. Understanding the overall trends of the data through numerical analysis is effective for interpreting the results. Particularly, clustering using MDS with Mahalanobis distance and principal component analysis with principal component scores clarifies the outlook of the analysis. In some cases, factor analysis and regression analysis are also effective and should be used in conjunction with QCA.